Neutrino Cosmology: neutrino mixing and BBN cosmology

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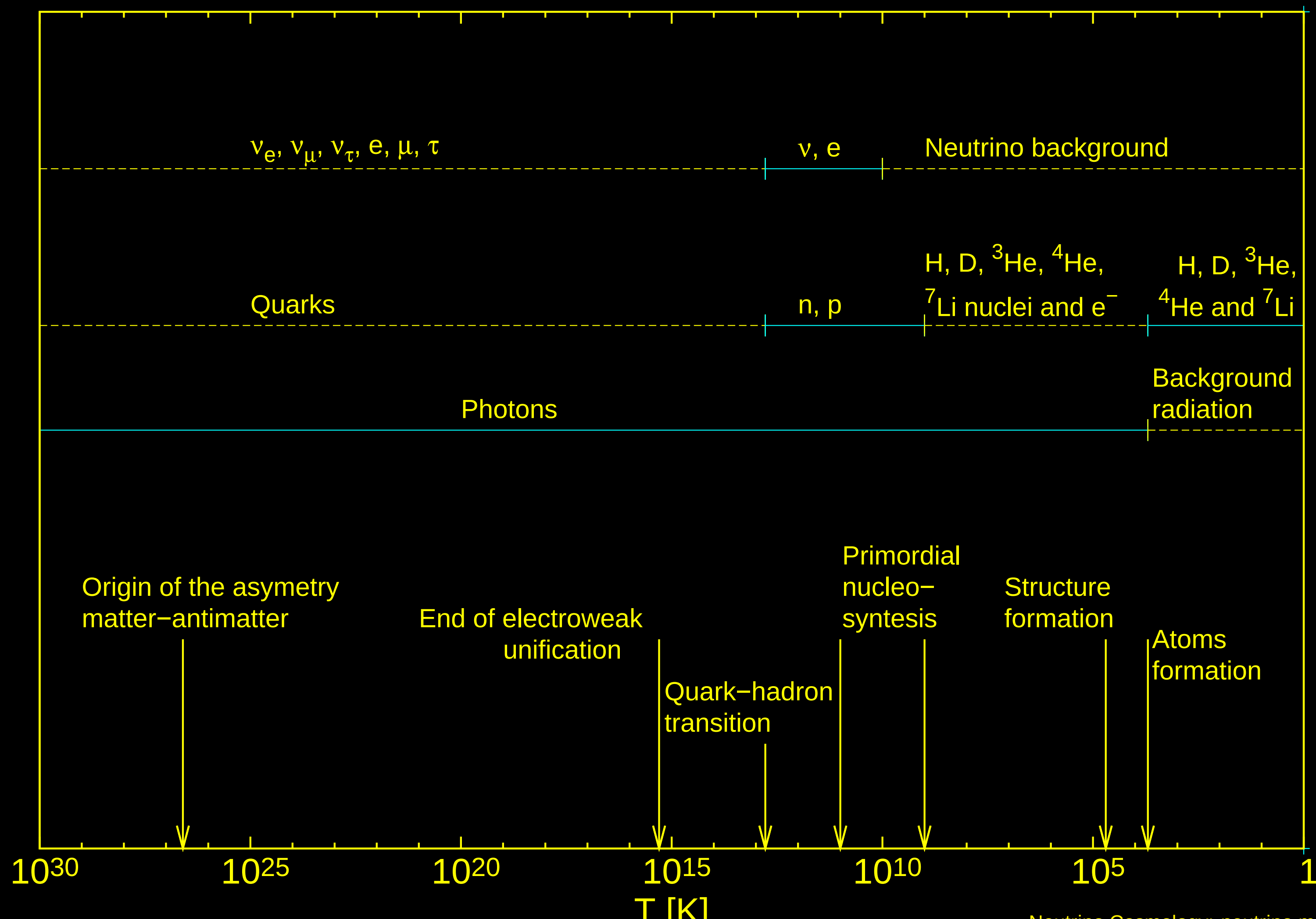
- Introduction to Big Bang Nucleosynthesis (BBN)
- Sterile neutrino in BBN
- Results
- Conclusions

Introduction

Thermal history of the Universe (Kolb and Turner, 1990)

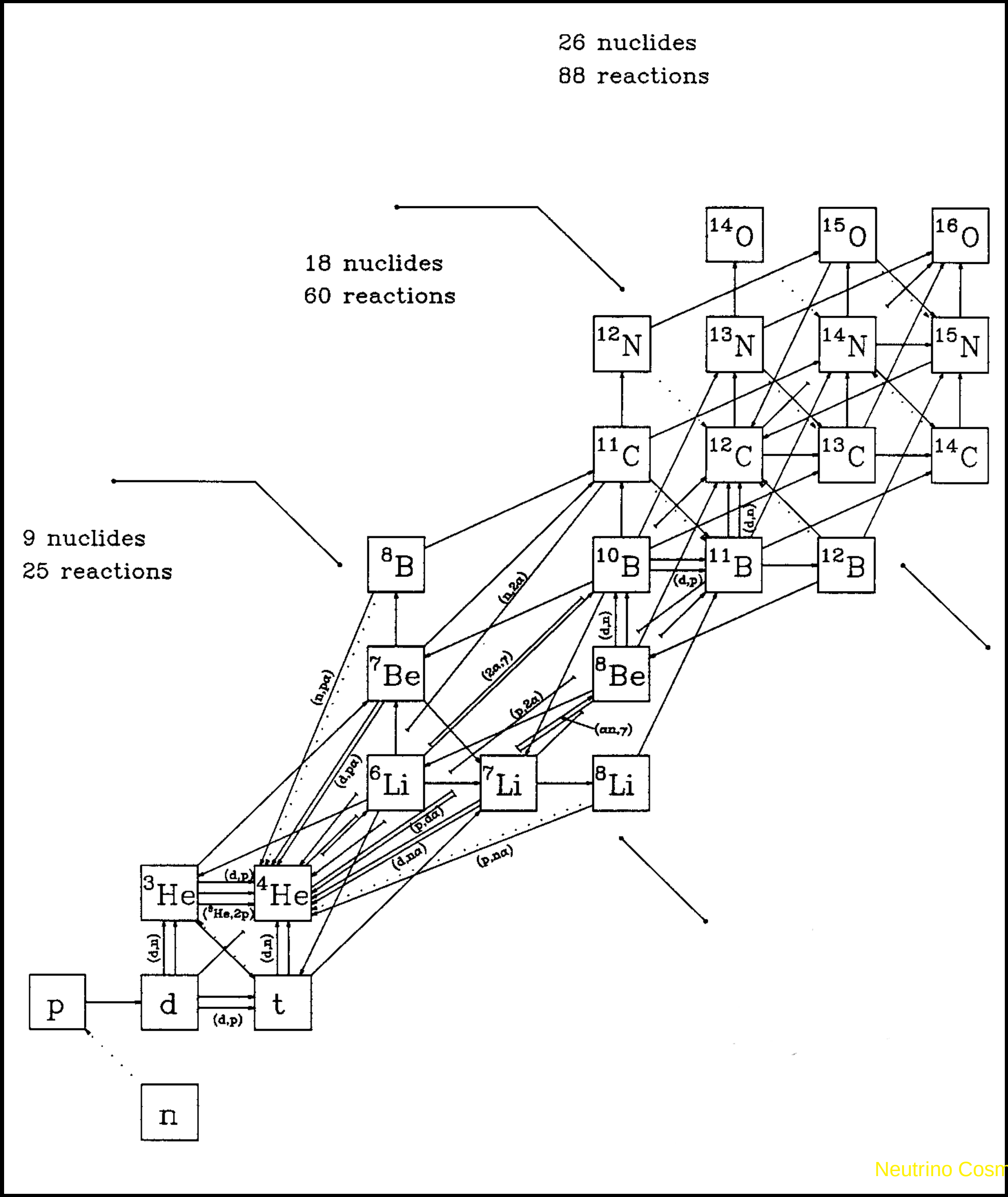


Introduction

- Big Bang Nucleosynthesis is the process where the light nuclei are formed.
- It takes place during the first three minutes after the Big Bang (temperatures of the order of 10^9 K).
- The model to obtain the primordial abundances has only one free parameter, the baryon density $\Omega_B h^2$.
- It's one of the most important tools to study one of the first stage of the Universe.

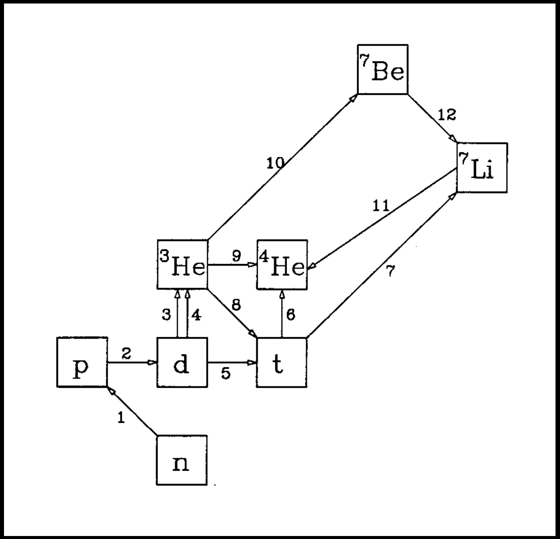
Introduction

(Kawano, 1992)



Introduction

Most important reactions that keep the thermal and chemical balance (Kawano, 1992)



1.- $n \leftrightarrow p$

2.- $n + p \leftrightarrow d + \gamma$

3.- $d + p \leftrightarrow ^3 \mathrm{He} + \gamma$

4.- $d + d \leftrightarrow ^3 \mathrm{He} + n$

5.- $d + d \leftrightarrow t + p$

6.- $t + d \leftrightarrow ^4 \mathrm{He} + n$

7.- $t + ^4 \mathrm{He} \leftrightarrow \gamma + ^7 \mathrm{Li}$

8.- $^3\mathrm{He} + n \leftrightarrow t + p$

9.- $d + ^3 \mathrm{He} \leftrightarrow ^4 \mathrm{He} + p$

10.- $^3\mathrm{He} + ^4 \mathrm{He} \leftrightarrow \gamma + ^7 \mathrm{Be}$

11.- $^7\mathrm{Li} + p \leftrightarrow ^4 \mathrm{He} + ^4 \mathrm{He}$

12.- $^7\mathrm{Be} + n \leftrightarrow p + ^7 \mathrm{Li}$

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Observational data of primordial abundances

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HII regions and extrapolation to zero metalicity

$(Y_{^4\mathrm{He}} = 0.2565 \pm 0.0010)$ (Izotov and Thuan, 2010)

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- ^{7}Li

Stars with low metalicity and extrapolation to zero metalicity

$\left(Y_{^7\mathrm{Li}} = (1.26 \pm 0.26) \times 10^{-10}\right)$ (Bonifacio et al., 2007)

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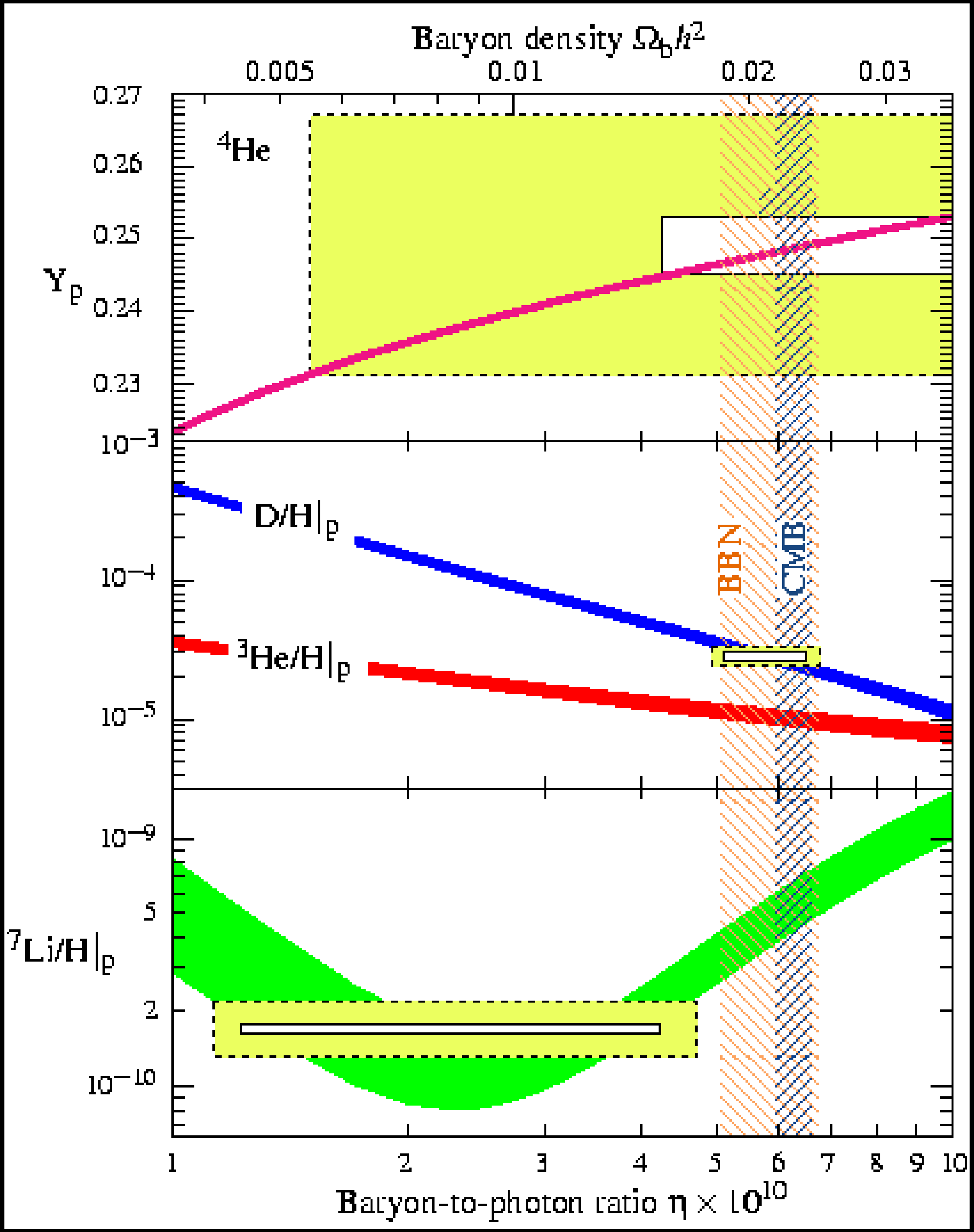
- The study of the Cosmic microwave background radiation (CMB) is one of the most important tools to obtain, with great accuracy, the cosmological parameters, such as the baryon density, dark matter density, etc.
- The value of the baryon density estimated from WMAP data is (Spergel et al, 2007):

$$\Omega_B h^2 = 0.0224 \pm 0.0009$$

(K. Nakamura et al. (Particle Data Group), 2010)



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- The observed value of $^{7}\mathrm{Li}$ is not in agreement with the theoretical value obtained with the cosmological parameters of WMAP
- The BBN model takes into account three massless neutrinos (no neutrino oscillation)
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- We studied the effects of the inclusion of massive and sterile neutrinos in the first three minutes of the Universe, upon the primordial abundances.

In particularly, we studied the following scenarios

- two massive neutrino states
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- We set constraints on the mixing parameters in order to reconcile the observations of primordial abundances with WMAP estimations

Neutrinos in cosmology

- We obtained the distribution functions for light neutrinos, considering an sterile neutrino, as a function of the mixing parameters ϕ and Δm^2
- We computed the reaction rates for the conversion of neutron to proton
- We calculated the primordial abundances as a function of ϕ and Δm^2
- We set constraints on the mixing parameters and the baryon density by the comparison of the theoretical results and the observations

- Normal hierarchy

$$\begin{pmatrix} \nu_l(t) \\ \nu_m(t) \\ \nu_h(t) \\ \nu_s(t) \end{pmatrix} = U \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \\ \nu_4(t) \end{pmatrix}$$

where

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} & 0 \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} & 0 \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & 0 & \sin\phi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\phi & 0 & 0 & \cos\phi \end{pmatrix}$$

s_{ij} (c_{ij}) stands for $\sin\theta_{ij}$ ($\cos\theta_{ij}$)

- Inverse hierarchy

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} & 0 \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} & 0 \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi & \sin\phi \\ 0 & 0 & -\sin\phi & \cos\phi \end{pmatrix}$$

- The equation for the density matrix (in the mass eigenstate basis), $\mathcal{F}_{ij} =< \nu_i|\nu_j >$, is

$$\left(\frac{\partial \mathcal{F}}{\partial t} - \mathrm{H}_{\mathrm{H}} E_\nu \frac{\partial \mathcal{F}}{\partial E_\nu}\right) = \imath \left[\mathcal{H}_0, \mathcal{F}\right]$$

where t is the time, H_{H} the universe expansion constant ($\mathrm{H}_{\mathrm{H}} = \mu_P T^2$), E_ν stands for the neutrino energy and $\mathcal{H}_0 = \mathrm{diag}\left(E_1, E_2, E_3, E_4\right)$

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- If one considered electron-neutrino interactions

$$\left(\frac{\partial \mathcal{F}}{\partial t} - \mathrm{H_H} E_\nu \frac{\partial \mathcal{F}}{\partial E_\nu}\right) = \imath \left[\mathcal{H}_0 + \sqrt{2} G_F \left(n_e(T) - \frac{8}{3M_W^2}\rho_e(T) E_\nu\right) A, \mathcal{F}\right]$$

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- The initial condition can be written as

$$\left(\begin{array}{cc} f_a & f_{as} \\ f_{sa} & f_s \end{array} \right) \Bigg|_{T_0} \quad = \quad \frac{1}{1 + e^{E_\nu / T_0}} \left(\begin{array}{cc} I & 0 \\ 0 & 0 \end{array} \right)$$

Neutrinos in cosmology

- Solutions
  - two states

$$f_l = \frac{1}{1+e^{E_\nu/T}}\left\{1+\frac{\sin^2 2\phi}{2}\left[\cos\left(\frac{\Delta m^2}{6\mu_P}\frac{T}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_0^3}\right)\right)-1\right]\right\}$$

  - three states, normal hierarchy

$$f_l = \frac{1}{1+e^{E_\nu/T}} + \frac{\cos^2\theta_{13}\cos^2\theta_{12}}{1+e^{E_\nu/T}}\frac{\sin^2 2\phi}{2}\left[\cos\left(\frac{\Delta m^2}{6\mu_P}\frac{T}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_0^3}\right)\right)-1\right]$$

  - three states, inverse hierarchy

$$f_l = \frac{1}{1+e^{E_\nu/T}}\left\{1+\sin^2\theta_{13}\frac{\sin^2 2\phi}{2}\left[\cos\left(\frac{\Delta m^2}{6\mu_P}\frac{T}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_0^3}\right)\right)-1\right]\right\}$$

- Reaction rates of $n \leftrightarrow p$

$$\lambda_{(\nu+n\rightarrow p+e^-)} = \frac{255}{4\tau\left(\delta m_{np}\right)^5}\int_0^\infty dp_\nu\; p_\nu E_\nu p_e E_e \left(1-f_e\right) f_l$$

$$\lambda_{(e^++n\rightarrow p+\overline{\nu})} = \frac{255}{4\tau\left(\delta m_{np}\right)^5}\int_0^\infty dp_e\; p_\nu E_\nu p_e E_e \left(1-f_l\right) f_e$$

$$f_e = \left(1+e^{E_e/T}\right)^{-1}$$

$$f_l = \left(1+e^{E_\nu/T}\right)^{-1}\left\{1-\frac{\sin^2 2\phi}{2}\xi\left[1-g\left(\Delta m^2, E_\nu, T\right)\right]\right\}$$

Primordial abundances

$$
\begin{aligned}
\dot{Y}_n &= Y_d Y_d R[dd; n^3\mathrm{He}] + Y_d Y_t R[dt; n^4\mathrm{He}] + Y_p Y_t R[pt; n^3\mathrm{He}] \\
&\quad + Y_d Y_\gamma R[d\gamma; np] - Y_n Y_p R[np; d\gamma] - Y_n Y_{^3\mathrm{He}} R[n^3\mathrm{He};\ tp] \\
&\quad - Y_n [n] \\
\dot{Y}_d &= Y_n Y_p R[np; d\gamma] - 2 Y_d Y_d \left(R[dd; pt] + R[dd; n^3\mathrm{He}] \right) \\
&\quad - Y_d Y_t R[dt; n^4\mathrm{He}] - Y_d Y_{^3\mathrm{He}} R[d^3\mathrm{He};\ p^4\mathrm{He}] \\
&\quad - Y_d Y_\gamma R[d\gamma; np] - Y_d Y_p R[dp;^3\mathrm{He}\,\gamma] \\
\dot{Y}_t &= Y_n Y_{^3\mathrm{He}} R[n^3\mathrm{He};\ pt] + Y_d Y_d R[dd; pt] - Y_d Y_t R[dt; n^4\mathrm{He}] \\
&\quad - Y_p Y_t R[pt; n^3\mathrm{He}] - Y_p Y_t R[pt; \gamma^4\mathrm{He}] \\
\dot{Y}_{^3\mathrm{He}} &= Y_d Y_p R[pd;^3\mathrm{He}\,\gamma] + Y_t Y_p R[pt; n^3\mathrm{He}] + Y_d Y_d R[dd; n^3\mathrm{He}] \\
&\quad - Y_d Y_{^3\mathrm{He}} R[d^3\mathrm{He};\ p^4\mathrm{He}] - Y_n Y_{^3\mathrm{He}} R[n^3\mathrm{He};\ pt]
\end{aligned}
$$

Primordial abundances

$$
\begin{aligned}
\dot{Y}_{^4\mathrm{He}} &= Y_d Y_{^3\mathrm{He}} R[d^3\mathrm{He};\ p^4\mathrm{He}] + Y_n Y_{^3\mathrm{He}} R[n^3\mathrm{He};\ ^4\mathrm{He}\,\gamma] \\
&\quad + Y_d Y_t R[dt; n^4\mathrm{He}] + Y_p Y_t R[pt; \gamma^4\mathrm{He}] \\
\dot{Y}_{^6\mathrm{Li}} &= Y_d Y_{^4\mathrm{He}} R[d^4\mathrm{He};\ ^6\mathrm{Li}\,\gamma] - Y_n Y_{^6\mathrm{Li}} R[n^6\mathrm{Li};\ ^4\mathrm{He}\,t] \\
&\quad - Y_p Y_{^6\mathrm{Li}} R[p^6\mathrm{Li};\ t^4\mathrm{He}] \\
\dot{Y}_{^7\mathrm{Li}} &= Y_n Y_{^4\mathrm{He}} R[n^6\mathrm{Li};\ ^7\mathrm{Li}\,\gamma] + Y_n Y_{^7\mathrm{Be}} R[n^7\mathrm{Be};\ p^7\mathrm{Li}] \\
&\quad + Y_t Y_{^4\mathrm{He}} R[t^4\mathrm{He};\ ^7\mathrm{Li}\,\gamma] - Y_p Y_{^7\mathrm{Li}} R[p^7\mathrm{Li};\ ^4\mathrm{He}\,^4\mathrm{He}] \\
&\quad - Y_n Y_{^7\mathrm{Li}} R[n^7\mathrm{Li};\ ^8\mathrm{Li}\,\gamma] \\
\dot{Y}_{^7\mathrm{Be}} &= Y_p Y_{^6\mathrm{Li}} R[p^6\mathrm{Li};\ ^7\mathrm{Be}\,\gamma] + Y_{^3\mathrm{He}} Y_{^4\mathrm{He}} R[^3\mathrm{He}\,^4\mathrm{He};\ ^7\mathrm{Be}\,\gamma] \\
&\quad - Y_\gamma Y_{^7\mathrm{Be}} R[^7\mathrm{Be}\,\gamma;^3\mathrm{He}\,^4\mathrm{He}] - Y_n Y_{^7\mathrm{Be}} R[n^7\mathrm{Be};\ p^7\mathrm{Li}] \\
&\quad - Y_p Y_{^7\mathrm{Be}} R[p^7\mathrm{Be};\ \gamma^8\mathrm{Li}] - Y_d Y_{^7\mathrm{Be}} R[d^7\mathrm{Be};\ ^4\mathrm{He}\,^4\mathrm{He}\,p]
\end{aligned}
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Primordial abundances

The equation for each nuclei can be written as

$$\dot{Y}_i = J(t) - \Gamma(t) Y_i$$

where $J(t)$ and $\Gamma(t)$ are the source and sink terms (Esmailzadeh et al, 1991).

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Quasi-static equilibrium (QSE)

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- Computation of the neutron abundance until the freezeout of the weak interaction
- Computation of the primordial abundances in stages characterized by the importance of the reactions. For the nucleus with mayor production rate one solves the differential equation $\dot{Y}_i = J(t) - \Gamma(t) Y_i$. For the other nuclei one solves the QSE equations

Primordial abundances

- Neutron abundance

$$\lambda_{np}(y) = \frac{255}{\tau}\left[1-\frac{\sin^2 2\phi}{4}\xi\right]\left(\frac{1}{y^3}+\frac{6}{y^4}+\frac{12}{y^5}\right)+ + \frac{255}{4\tau}\frac{\sin^2 2\phi}{2}\xi\int_0^\infty dq\; q^2\,(q+1)^2\,e^{-qy} g\left(\Delta m^2, q, y\right)$$

where $y = \frac{\delta m_{np}}{T}$
Neutron equilibrium equation

$$\frac{dX_n}{dt} = \lambda_{pn}(t)(1-X_n(t)) - \lambda_{np}(t)X_n(t)$$

where t is the time and $X_n(t)$ is the neutron to baryon ratio. The solution results (Bernstein et al., 1989)

$$X_n = \int_0^\infty dw\, \frac{e^w}{(1+e^w)^2} e^{-\left(\mu_P(\delta m_{np})^2\right)^{-1}\int_w^\infty du\; u\left(1+e^{-u}\right)\lambda_{np}(u)}$$

- Primordial abundances

$$\frac{dY_i}{dt} = J(t) - \Gamma(t) Y_i$$

- χ^2 test to obtain the best-fit parameters

$$\chi^2 = \sum \frac{\left(Y_x^{obs} - Y_x^{teo}\left(\Omega_B h^2, \sin^2 2\phi, \Delta m^2\right)\right)^2}{\sigma_x^2}$$

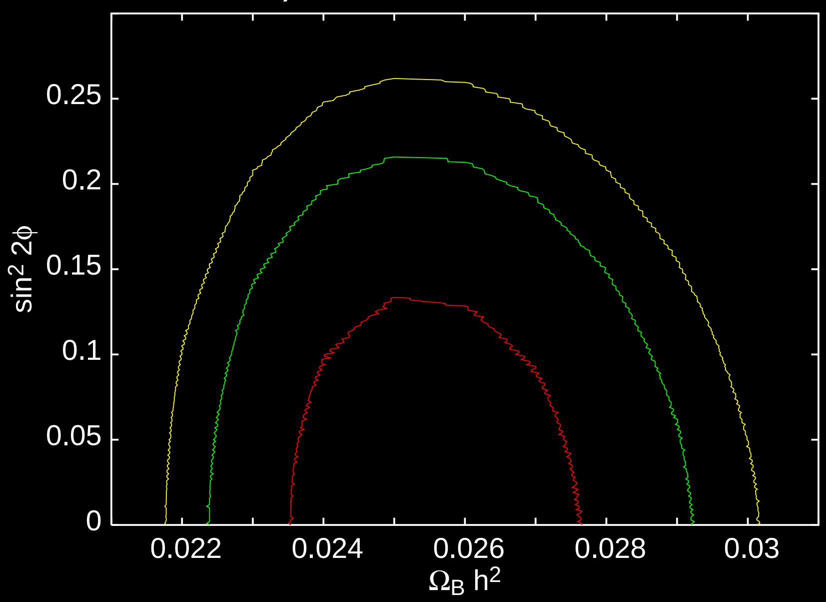
Results

$\Delta m^2 = 10^{-8}\,\mathrm{eV}^2$ (Civitarese and Mosquera, IJMPE 17, 351 (2008), Civitarese and Mosquera, PRC 77, 05806 (2008))

	All data		Excluding $^7\mathrm{Li}$	
	$\sin^2 2\phi \pm \sigma$	$\Omega_B h^2 \pm \sigma$	$\sin^2 2\phi \pm \sigma$	$\Omega_B h^2 \pm \sigma$
Two states	0.002 ± 0.033	0.0250 ± 0.0010	$0.221^{+0.095}_{-0.092}$	0.0230 ± 0.0020
Two states + int	0.007 ± 0.074	0.0250 ± 0.0020	$0.196^{+0.048}_{-0.084}$	0.0230 ± 0.0020
Three states	0.000 ± 0.026	0.0253 ± 0.0015	0.018 ± 0.098	0.0216 ± 0.0017

$$\left(\Omega_B h^2\right)_{WMAP} = 0.0224 \pm 0.0009$$

Two states, all data



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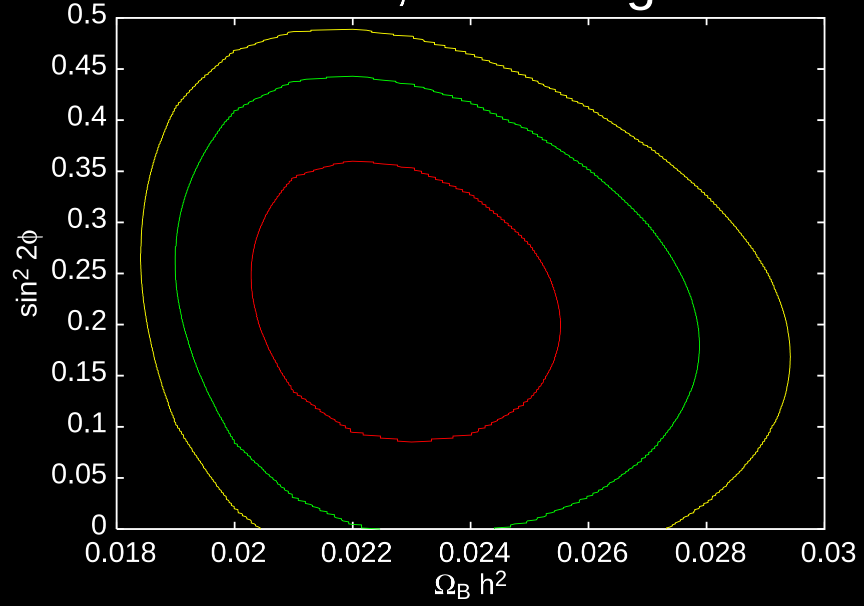

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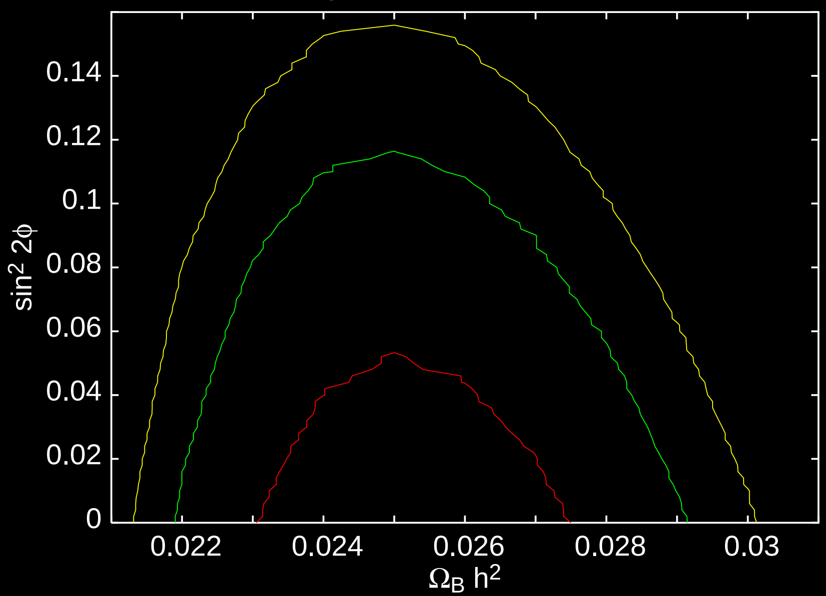

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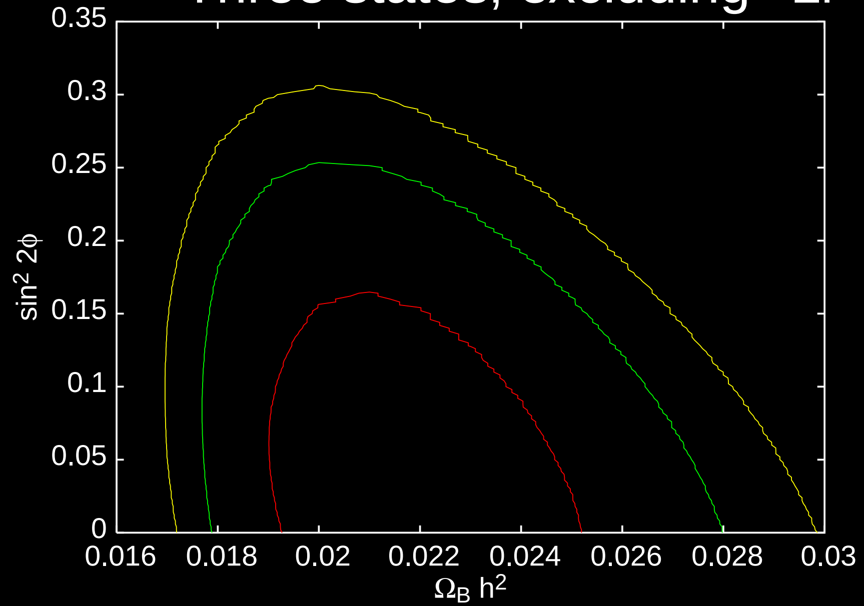

Conclusions

The compatibility analysis between the theoretical and observational primordial abundances, shows that

- there exists sensitivity with the mass hierarchy and with the mixing parameters between active and sterile neutrinos
- there exists sensitivity of the baryon density with respect to the mixing angle
- $\Omega_B h^2$ and $\sin^2 2\phi$ are in good agreement with the WMAP results and LSND data when $^7\mathrm{Li}$ is excluded in the analysis