A light $3 + 1$ sterile neutrino model study of MINOS experiment

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Outline

Part I

- Goodness of fit tests

Part II

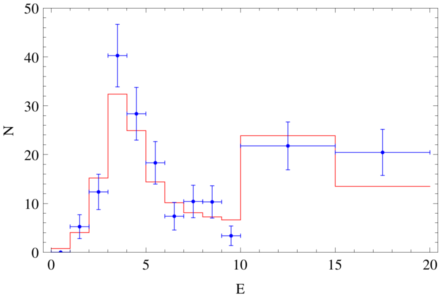
- Neutrino oscillation and the mass eigenstates
- MINOS overview
- Results ($\nu_\mu + \bar{\nu}_\mu$)
  - Effective 2ν model
  - Light $3 + 1$ model
- Conclusions
- Perspectives

Part I

Goodness of fit tests

Goodness of fit tests

- Is the following fit reasonable?



General Gaussian χ^2 :

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(N_{\mathrm{e}}^{(i)} - N_{\mathrm{o}}^{(i)}\right)^2}{\sigma_i^2}$$

$N_{\mathrm{e}} \rightarrow$ expected

$N_{\mathrm{o}} \rightarrow$ observed

Best fit : set of parameters in N_e that minimizes χ^2.

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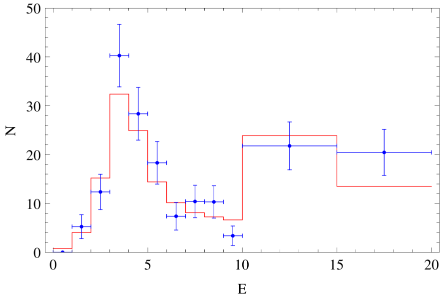

General Gaussian χ^2 :

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(N_e^{(i)} - N_o^{(i)}\right)^2}{\sigma_i^2}$$

$N_e \rightarrow$ expected
$N_o \rightarrow$ observed

Best fit : set of parameters in N_e that minimizes χ^2.

- There is a **systematic** way to decide that: standard goodness of fit (SG) .

  Does **not** requires comparison to other fits.

Goodness of fit tests

- $\chi^2_{\text{min}}/\text{d.o.f.}$ is a good test?
  - $\chi^2_{\text{min}} = 0$ (perfect fit)
  - $\chi^2_{\text{min}} = \text{d.o.f.}$ (**good**, valid for Gaussian f_{χ^2})
  - $\chi^2_{\text{min}} \to +\infty$ (worst case)

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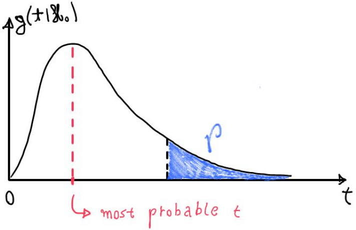
- We need a test that
  - does not requires Gaussian χ^2 p.f.d.;
  - is more sensitive (varies in a shorter interval).

- Ideal quantity for such task: p-value

$$p \equiv \int_{t_{\rm obs}}^{+\infty} dt\, g\left(t|H_0\right)$$



Goodness of fit tests

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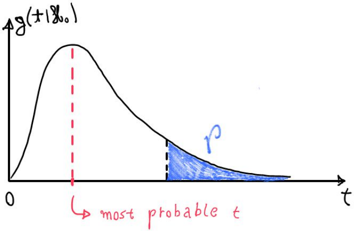
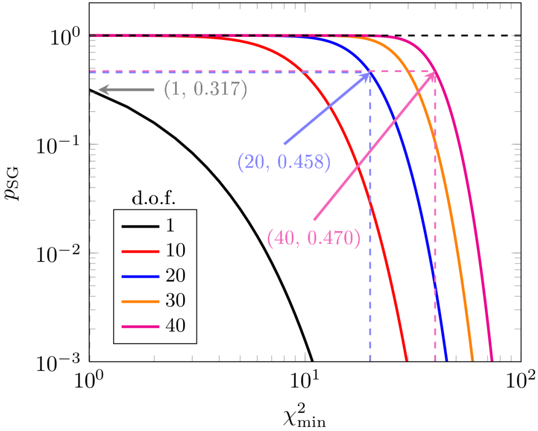


- Standard g.o.f. (SG) : $t = \chi^2$.
  - $p = 1$ (perfect fit)
  - $p = 0.5$ (**good**, valid for Gaussian f_{χ^2})
  - $p = 0$ (worst case)

SG – p-value

$$p_{\mathrm{SG}} \equiv \int_{\chi^2_{\min}}^{+\infty} dt\, f_{\chi^2}\left(t, N-P\right)$$



- For $(N-P) \gtrsim 10^2$:

  $$\frac{\chi^2_{\min}}{\mathrm{d.o.f.}} \approx 1 \pm \sqrt{\frac{2}{N-P}}$$

- Tendency:

  $$\frac{\chi^2_{\min}}{\mathrm{d.o.f.}} \to 1 \qquad p_{\mathrm{SG}} \to 0.5$$

- Within 1σ of f_{χ^2}:

  $$p_{\mathrm{SG}} \approx 0.5 \pm 0.34$$

Goodness of fit tests

- Furthermore, we wish a test capable of measuring the compatibility of distinct data sets

parameter g.o.f. (PG) : $t = \bar{\chi}^2 \equiv \sum \Delta\chi_j^2$

$$p_{\mathrm{PG}} \equiv \int_{\bar{\chi}^2_{\mathrm{min}}}^{+\infty} dt\, f_{\chi^2}\left(t, P_c \equiv \sum_r P_r - P\right)$$

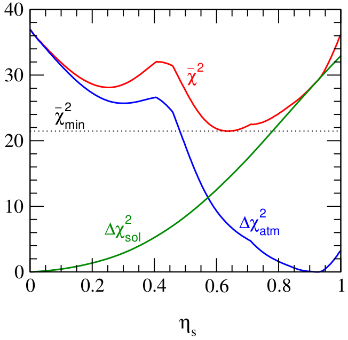
following Maltoni & Schwetz (2003)[†].

$P_r \to$ # parameters of dataset r
$P \to$ # parameters (overall)
$P_c \to$ # common parameters to all datasets

[†] M. Maltoni & T. Schwetz, Phys. Rev. D **68**, 0033020 (2003).

Goodness of fit tests

Data	**SG**	**PG**
Sol+atm	0.783	3.538×10^{-6}
React+sol	0.980	0.937



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Part II

Neutrino oscillation and the mass eigenstates

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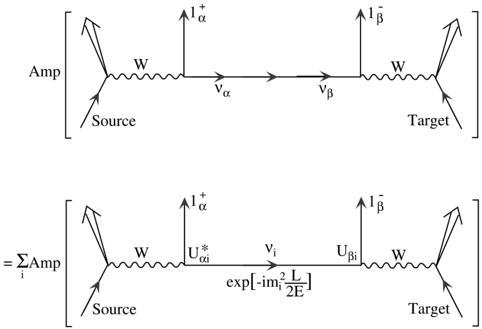
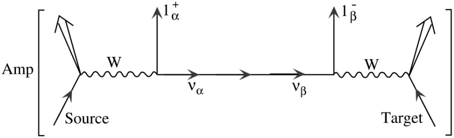
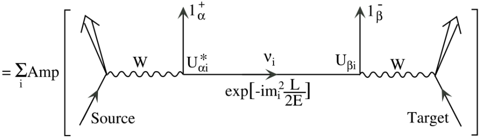


Boris Kayser, Springer Tracts in Modern Physics, v. 190 (2003).

Neutrino oscillation and the mass eigenstates





- U: neutrino mixing matrix . Example: 3 families

$$U \equiv \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \qquad \nu_\alpha^{(\mathrm{f})} = U_{\alpha i} \nu_i^{(\mathrm{m})}$$

Boris Kayser, Springer Tracts in Modern Physics, v. 190 (2003).

Neutrino oscillation and the mass eigenstates

- Survival probability:

$$
\begin{aligned}
P_{\nu_\alpha \to \nu_\alpha}(E, L) &= |\mathrm{Amp}\,[\nu_\alpha \to \nu_\alpha]|^2 \\
&= 1 - 4\, U_{aj}\, U^\dagger_{ja}\, U_{ak}\, U^\dagger_{ka}\, \delta_{a\alpha} \sin^2\left(\frac{\Delta m^2_{jk} L}{4E}\right)\Bigg|_{j>k}
\end{aligned}
$$

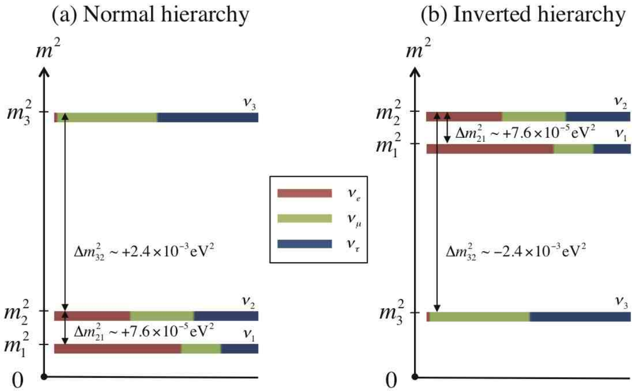
- U is usually parametrized in terms of rotations through angles called mixing angles .

Most simple example: 2 families

$$
U \equiv \begin{pmatrix} U_{e1} & U_{e2} \\ U_{o1} & U_{o2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.
$$

$$
P_{\nu_e \to \nu_e}(E, L) = 1 - \sin(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right).
$$

Neutrino oscillation and the mass eigenstates



$\Delta m^2_{32} \sim 10^{-3}$ eV^2: atmospheric scale

$\Delta m^2_{21} \sim 10^{-5}$ eV^2: solar scale (Nu-fit update: $7.45^{+0.19}_{-0.16} \times 10^{-5}$ eV^2)†

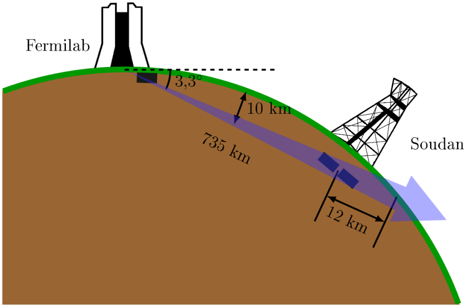
† Nu-fit collab., JHEP 12 (2012) 123 [arXiv:1209.3023].
K. Abe et al., arXiv:1109.3262 [hep-ex] (2011).

MINOS overview

MINOS overview

- Main Injector Neutrino Oscillation Search (MINOS).
- Accelerator neutrinos: Main Injector (NuMI) in Fermilab.
- 2 detectors: near & far. $L = 735$Km.
- Study of the region of parameters indicated by atmospheric neutrino experiments.

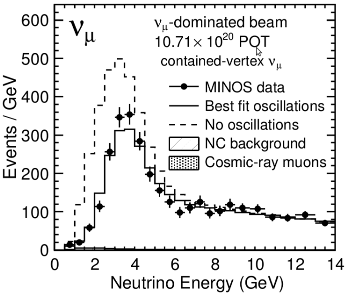
MINOS overview



MINOS collab. thesis: "Investigação de Mecanismos Alternativos a Oscilação de Neutrinos no Experimento MINOS", J. A. B. Coelho (2012).

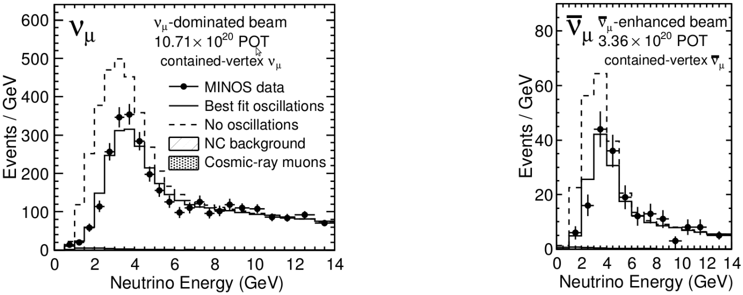
Results

Results – Effective 2ν model



MINOS collab., PRL **110**, 2518011 (2013).

Results – Effective 2ν model



- **Expected count** (N_e), from the survival probability and the number of no-oscillation events in the j-th bin:

$$N_\mathrm{e}^{(j)} \approx N_\text{no osc.}^{(j)} \times \frac{1}{\delta E} \int_{E_j - \frac{\delta E}{2}}^{E_j + \frac{\delta E}{2}} dE\, P_{\nu_\mu \to \nu_\mu}(E)$$

MINOS collab., PRL **110**, 2518011 (2013).

Results

- Already included in $N^{(j)}_{\text{no osc.}}$:

  energy resolution, neutrino flux, cross section, detector efficiency and near/far detector correlation.

- **Aproximation:** $N^{(j)}_{\text{no osc.}}$ does not vary much within a bin of energy.

Results

- General recipe for including "nuisance parameters":

$$\sigma_a \rightarrow \text{normalization}$$
$$\sigma_b \rightarrow \text{neutral current contamination}$$

$$N_{\mathrm{e}} \rightarrow (1+a)N_{\mathrm{e}} + (1+b)N_{\mathrm{BG}}$$
$$N_{\mathrm{o}} \rightarrow N_{\mathrm{o}} \quad \text{(background has \textbf{not} been subtracted)}$$
$$\chi^2 \rightarrow \chi^2 + \frac{a^2}{\sigma_a^2} + \frac{b^2}{\sigma_b^2}, \qquad \sigma_a = 1.6\% \qquad \sigma_b = 20\%.$$

Results

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- As a result:

$$\chi^2 = \sum_i \frac{\left[(1+a)N_\mathrm{e} + (1+b)N_\mathrm{BG} - N_\mathrm{o}^{(i)}\right]^2}{\sigma_i^2} + \frac{a^2}{\sigma_a^2} + \frac{b^2}{\sigma_b^2}$$

Results – Effective 2ν model

- Survival probability of ν_μ's:

$$P_{\nu_\mu \to \nu_\mu} = 1 - \sin^2(2\theta)\sin^2\left(\frac{1.267\Delta m^2[\mathrm{eV}^2]L[\mathrm{Km}]}{E[\mathrm{Gev}]}\right)$$

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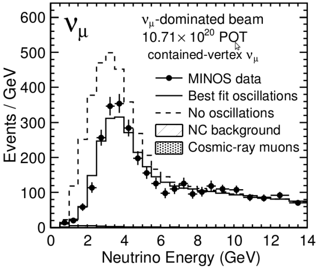
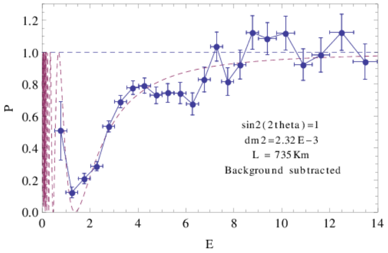
- For a qualitative discussion: $P \sim \dfrac{N_{\mathrm{osc}}}{N_{\mathrm{no-osc}}}$:

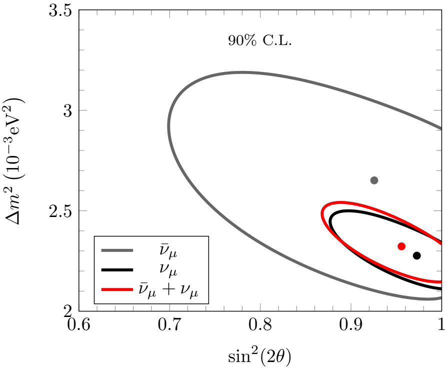


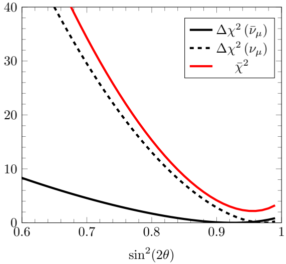


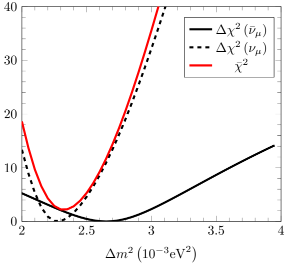
MINOS collab., PRL **110**, 2518011 (2013).

Results – Effective 2ν model

Pearson : $\chi^2 = \sum_i \frac{\left(N_e^{(i)} - N_o^{(i)}\right)^2}{N_e^{(i)}}$







Results – Effective 2ν model

- What is marginalization ?

In general, $\chi^2\,[n \text{ var.}] \rightarrow \chi^2\,[(n-1) \text{ var.}] \rightarrow \cdots$

Results – Effective 2ν model

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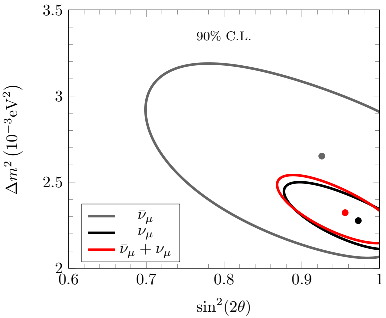
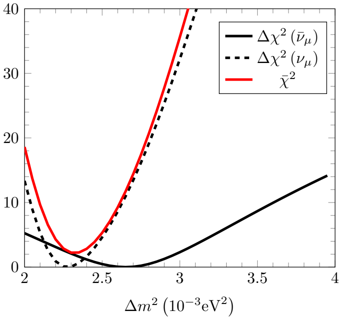
Example: $\bar{\chi}^2\left[\Delta m^2, \sin^2(2\theta)\right] \to \bar{\chi}^2\left(\Delta m^2\right)$

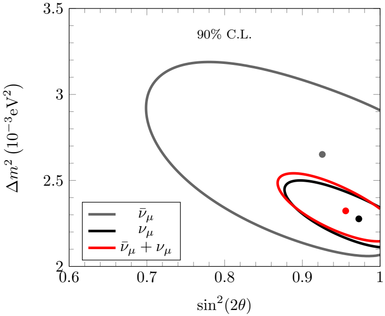




Results – Effective 2ν model

Data	$(N-P)$	$\chi^2_{\min}$	p_{SG}	Δm^2 $(\mathrm{eV}^2 \times 10^{-3})$	$\sin^2(2\theta)$
ν_μ	$23-2=21$	25.431	0.229	$2.28^{+0.09}_{-0.08}$	$0.97^{+0.03}_{-0.04}$
$\bar{\nu}_\mu$	$12-2=10$	8.684	0.562	2.7 ± 0.2	$0.93^{+0.07}_{-0.09}$
$\nu_\mu + \bar{\nu}_\mu$	$23+12-2=33$	36.287	0.318	0.23 ± 0.01	$0.96^{+0.03}_{-0.04}$

Data	P_c	$\bar{\chi}^2_{\min}$	p_{PG}
$\nu_\mu + \bar{\nu}_\mu$	$4-2=2$	2.173	0.337



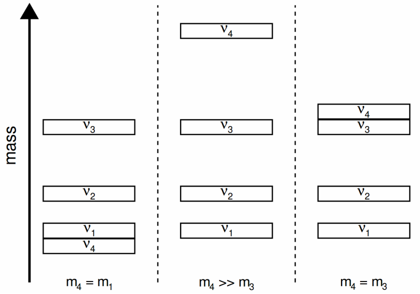
Results – Light 3 + 1 model

- Mass hierarchy: $m_3 > m_2 \gtrsim m_1$

$$\Delta m^2_{32} \sim 10^{-3}\,\mathrm{eV}^2$$

- What about m_4?

$$\Delta m^2_{21} \sim 10^{-5}\,\mathrm{eV}^2$$



MINOS collab., PRD **81**, 052004 (2010)

Results – Light 3 + 1 model

$$
\begin{aligned}
P_{\nu_\mu \to \nu_\mu}(E, L) \;\approx\;& 1 - 4\,|U_{\mu 4}|^2 \left(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \\
& -4\;|U_{\mu 4}|^2\,|U_{\mu 3}|^2 \sin^2\left[\left(\Delta m^2_{41} - \Delta m^2_{32}\right)\frac{L}{4E}\right] \\
& -4\,|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)
\end{aligned}
$$

- We assume:

$$\Delta m^2_{32} = 2.41 \times 10^{-3}\mathrm{eV}^2$$

$$10^{-3}\,\mathrm{eV}^2 \leq \Delta m^2_{41} \leq 1\,\mathrm{eV}^2$$

Results – Light 3 + 1 model

- For MINOS, the scale of mass that has influence under the oscillation is between 10^{-3} eV^2 and 10^{-2} eV^2:

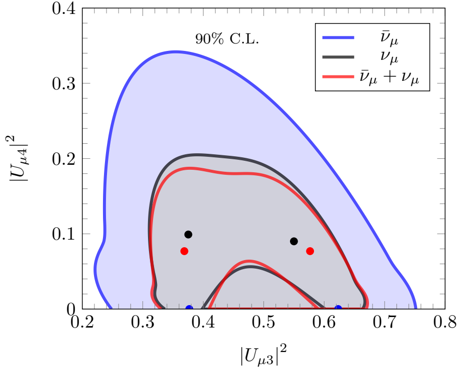
$$1.267 \frac{\Delta m^2 \left[\mathrm{eV}^2\right] L \left[\mathrm{Km}\right]}{E \left[\mathrm{Gev}\right]} \sim \frac{\pi}{2}$$

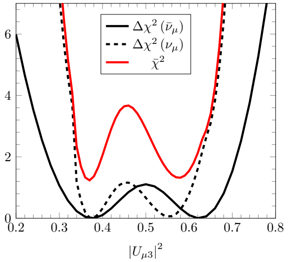
$$1.267 \frac{\Delta m^2 \left[\mathrm{eV}^2\right] 735 \left[\mathrm{Km}\right]}{1 \left[\mathrm{Gev}\right]} \sim \frac{\pi}{2} \quad \Rightarrow \quad \Delta m^2 \sim 10^{-3} \mathrm{eV}^2$$

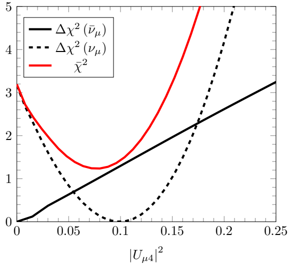
$$1.267 \frac{\Delta m^2 \left[\mathrm{eV}^2\right] 735 \left[\mathrm{Km}\right]}{14 \left[\mathrm{Gev}\right]} \sim \frac{\pi}{2} \quad \Rightarrow \quad \Delta m^2 \sim 10^{-2} \mathrm{eV}^2$$

Results – Light 3 + 1 model

$$\text{Pearson}: \chi^2 = \sum_i \frac{\left(N_e^{(i)} - N_o^{(i)}\right)^2}{N_e^{(i)}}$$







Results – Light 3 + 1 model

- Why 2 minima?
- 4 are expected, because of the quadratic terms in the variables $|U_{\mu 3}|^2$ and $|U_{\mu 4}|^2$:

$$
\begin{aligned}
P_{\nu_\mu \to \nu_\mu}(E, L) \approx\ & 1 - 4\,|U_{\mu 4}|^2 \left(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) \\
& - 4\ |U_{\mu 4}|^2\,|U_{\mu 3}|^2 \sin^2\left[\left(\Delta m^2_{41} - \Delta m^2_{32}\right) \frac{L}{4E}\right] \\
& - 4\,|U_{\mu 3}|^2 \left(1 - |U_{\mu 3}|^2 - |U_{\mu 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)
\end{aligned}
$$

- 2 of them are excluded: $\Delta m^2_{32} \sim 10^{-3}\text{eV}^2$ requires small $|U_{\mu 4}|^2$.

Results – Light 3 + 1 model

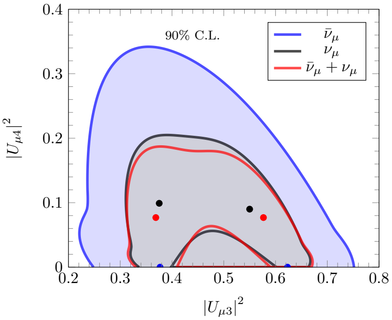
Data	$(N-P)$	$\chi^2_{\min}$	p_{SG}	$\lvert U_{\mu3}\rvert^2$	$\lvert U_{\mu4}\rvert^2$
ν_μ	$23-2=21$	24.146	0.286	$0.38^{+0.10}_{-0.03}$	0.10 ± 0.05
		24.203	0.283	0.6 ± 0.1	||
$\bar{\nu}_\mu$	$12-2=10$	9.782	0.460	0.4 ± 0.1	< 0.08
		||	||	$0.62^{+0.06}_{-0.09}$	||
$\nu_\mu + \bar{\nu}_\mu$	$23+12-2=33$	35.162	0.366	$0.37^{+0.04}_{-0.03}$	$0.08^{+0.05}_{-0.06}$
		35.245	0.362	$0.6^{+0.1}_{-0.2}$	||

Data	P_c	$\bar{\chi}^2_{\min}$	p_{PG}
$\nu_\mu + \bar{\nu}_\mu$	$4-2=2$	1.234	0.540
		1.317	0.518

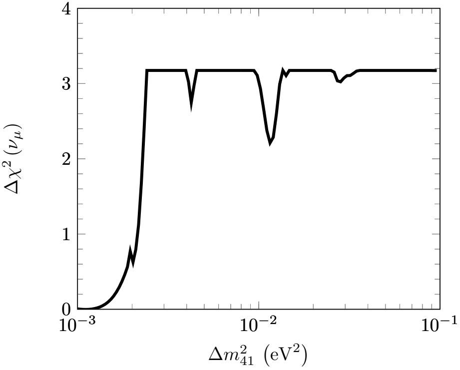
Remember: for 2ν, $p_{\mathrm{PG}} = 0.337$.

$\lvert U_{\mu3}\rvert^2$	$\sin^2(2\theta^{\mathrm{atm}}_{\mu\mu})$
0.37	0.9324
0.6	0.96

$$\sin^2 2\theta^{\mathrm{atm}}_{\mu\mu} \equiv 4\,\lvert U_{\mu3}\rvert^2 \left(1 - \lvert U_{\mu3}\rvert^2\right)$$



Results – Light 3 + 1 model



$\Delta m^2_{41} < 0.002\,\mathrm{eV}^2$

Conclusions

We have

- reproduced MINOS confidence regions for 3 ν (effective 2 ν);
- studied the level of compatibility (PG) between ν and $\bar{\nu}$ data;
- stablished the confidence regions for $|U_{\mu 3}|^2$ and $|U_{\mu 4}|^2$ for a 3+1 light sterile model and
- verified that PG has been enhanced (better compatibility than with 3ν) and
- made a preliminary study of Δm^2_{41}, verifying multiple local minima.

Perspectives

- Investigate further $\chi^2 \times \Delta m^2_{41}$;
- introduction of **matter effects** and
- indirect effect of sterile neutrinos (through oscillation) on **neutral current** event count.[†]

[†] MINOS collab., PRL **107**, 011802 (2011).

Acknowledgements

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